

THE STAGING OF VENTURE CAPITAL FINANCING:
MILESTONE VS. ROUNDS*

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Abstract

Venture capital funding is commonly provided to start-up firms on a piecemeal basis over several stages. One way in which this can be implemented is through milestone financing, where a venture capitalist commits upfront to providing additional future funding contingent upon the firm meeting certain conditions, or milestones. Alternately, the firm can operate without a firm commitment in place, still reasonably expecting to be able to receive additional rounds of funding after goals are met (round financing).

We identify four dimensions which can affect the optimal contract and choice of financing method: entrepreneurial effort, venture capitalist effort, venture capitalist preference for liquid investments, and heterogeneous expectations about the feasibility of the underlying real technology. The effects of these on the optimal milestone and round financing contracts are examined. Firms that prefer milestone financing to round financing (and conversely) are characterized.

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1. Introduction

It is now difficult to envision grooming of technology-based start-up firms without venture capital backing. From an academic perspective, two features of venture capital are of particular interest compared to other forms of financing. First, venture capital investment is often called smart money, denoting the fact that it plays a dual role. In addition to providing funding, venture capitalists serve their portfolio firms by providing coaching and guidance, as well as networking for strategic alliances and for further funding. Second, unlike investments in quoted companies, there are only a few investors involved in the funding, all of whom are presumed to be sophisticated. Therefore the terms of the funding need not be simple. In fact, they tend to be quite complicated so as to best address the various aspects of each particular case. Even a casual inspection of a typical term sheet reveals a strikingly large number of features, such as convertible and preferred securities, warrants, staged investment with milestones, anti-dilution ratchets, voting arrangements, liquidation preferences, and vesting arrangements.¹

The current study takes a close look at one phenomenon - the venture capitalist contractual commitment over time to the investment. Two types of financial arrangements are contrasted. The first is milestone financing, which includes both an immediate funding by the venture capitalist and a commitment for an additional investment later. The future funding commitment is at a predetermined price and is received by the start-up company once pre-specified technological or operational milestones are met. The second arrangement is round financing, in which there is no pre-commitment to invest beyond the current funding needs. Therefore, any subsequent investment is priced based on the realization and the status of the start-

¹ There are clearly other, non-financial, attributes that are unique to investment in start-ups. Most notably, they include the extremely high level of uncertainty (technological, managerial, and untested demand in the market), practically no history to analyze, early dependency on the entrepreneur, and lack of tangible assets. These factors intensify the need to use complicated financial contracts.

up company at the time of the subsequent round. We refer to the former arrangement as milestone investment, and the latter as round investment. The paper will characterize the situations where the use of milestones is better, vis-à-vis the circumstances where round investment is superior. In the process we explicitly account for several other key characteristics of investment in start-ups - the effort level of the entrepreneur, and the degree of involvement expended by the venture capitalist. Other pervasive features that our model accommodates are differential beliefs about the likelihood success of the start-up firm (the entrepreneur is often more optimistic) and the possibility that the venture capitalist has a preference for liquid investments. As will be elaborated upon later, venture capital funds have a strong incentive to exit their investment sooner rather than later, which we phrase liquidity preference. All these considerations play a role in the relative attractiveness of milestone financing compared to round financing. For instance, whether the entrepreneurial effort becomes more or less important when technology succeeds can make a pivotal difference to the optimal financial arrangement.

In a comprehensive study, Kaplan and Strömberg (2003) document the features of venture capital contracts. They analyze the terms sheets of over 200 rounds of venture backed investments and link the statistics to agency problems. They also list numerous types of observable and verifiable contingencies that are used in venture capital contracts. Sahlman (1990) and Lerner (1995) provide evidence on the dual role of venture capitalists and their involvement in monitoring and governance. Subsequently, Hellman and Puri (2000, 2002) statistically confirm that the in-kind services of venture capitalists are of economic significance, through a reduction in time to bring a product to market and by professionalizing the start-up company. Also on the empirical side, Gompers (1995) provides detailed statistics on staging of venture capital investments and explores factors that influence the amount invested in a round and the duration between rounds.

From an analytical perspective, Schmidt (2003) and Repullo and Suarez (2004) model the advisory role of the venture capitalists within a double-sided moral hazard framework which gives rise to features in convertible securities used in venture capital financing. Cornelli and Yosha (2003) show that the use of convertible securities mitigates the incentive of entrepreneurs to engage in window dressing practices.² The rationale for the advisory role of venture capitalist is analyzed in Casamatta (2003). Under moral hazard, if the entrepreneurial effort is more efficient (less costly) than that of the consultant, the latter is not hired unless invested financially in the project, in the spirit of venture capital involvement. Chemmanur and Chen (2003) study an opposite paradigm whereby pure financing (angel investing) is contrasted with active involvement of venture capital investment. The desired form of financing is characterized based on factors such as scarcity of venture capital funding. Along similar lines, Leshchinskii (2002) contrasts angel investing to venture capital financing by assuming that venture capitalists also aim to benefit from the interaction among their investments.

The advantage of staged financing is pointed out in Neher (1999) who shows that as human capital is gradually transformed to physical capital, the venture increases the value of its collateral, hence makes outside financing more affordable. Staging should coincide with significant economic developments in the enterprise. Another motivation for staging is provided by Fluck, Garrison and Myers (2004) who show that a commitment to syndicate financing in later stages reduces the entrepreneur's underprovision of effort. The current paper adds to this literature by incorporating the arguably key distinguishing feature of venture capital investment – hands on involvement, as well as the inherent mechanisms of stage financing. As it turns out, much of the results depend on the relative importance of the entrepreneur's effort compared to the in-kind contribution of the venture capitalist.

² Other analytical models that rationalize the use of convertible preferred stocks over a mix of debt and equity are Bascha (2001) and Houben (2002). Chang and Qiu (2003) compare alternative forms of preferred equity used in venture financing.

In comparing milestone financing with round financing, we identify four primary differences, giving rise to a relative advantage for each form of financing under certain conditions. First, because each round is contracted separately, round financing implies that the price of corporate claims sold at each financing stage should be set at a competitive level (conditional on common knowledge at that point in time). In contrast, milestone financing contracts multiple stages of financing simultaneously, giving the two contracting parties the ability to adjust the relative prices, and therefore the magnitudes, of the parties' claims on the firm across different outcomes. This may be advantageous in generating better incentives for the contracting parties to undertake costly effort to increase the value of the firm. In particular, if either the entrepreneur or venture capitalist can undertake private effort to increase corporate cash flows, it is well known that the principal-agent problem cannot result in first-best effort being undertaken (except in the extreme case where the party owns the rights to all marginal increases in corporate cash flows). Thus, in the context of entrepreneurial and/or venture capitalist effort, milestone financing can offer a valuable flexibility unavailable with round financing.

Second, the level of upfront commitment differs between milestone financing and round financing. At the first stage, round financing achieves a financing commitment for that stage alone. Milestone financing, in contrast, raises capital for the first stage, with the promise of additional funds in the future (conditional on achievement). Since the financier accepts a higher level of financing commitment under milestone financing, she should receive a commensurately larger claim on corporate cash flows, while the entrepreneur has a smaller claim on corporate cash flows. Again, in the context of effort that can be undertaken by the contracting parties, milestone and round financing result in different incentives for those parties to undertake effort.

Third, we consider the effects of heterogeneity of expectations about the likelihood of success of the real technology underlying the start-up firm. Of all possible real projects that could be undertaken, those expected to be most profitable should be taken on. Thus, an entrepreneur

undertaking a start-up will naturally be optimistic about his probability of success, perhaps even more so than their financiers. This gives rise to differential beliefs about the likelihood of various outcomes. Indeed, one of the objectives achieved through venture capital term sheets is to accommodate more optimistic entrepreneurs by committing to grant them more shares if promises are fulfilled and milestones are met (Kaplan and Strömberg 2003). Naturally, an optimal contract will tend to tilt toward giving contingent claims over possible states to the party who places the highest probability on that state.³

Fourth, we consider the effect of liquidity. Because venture capitalists raise their funds through limited partnerships which have a finite time horizon, they strongly prefer an exit from each investment before the end of the fund life. Venture capitalists also report performance in terms of IRR and so they prefer to return money to investors earlier than later. States which result in a possible sale or public offering (or other liquifying event) of the firm will be of particular interest for the venture capitalist. This has an effect similar to heterogeneity of beliefs: an optimal contract will tend to tilt toward giving the venture capitalist contingent claims in states leading to a liquidity event for the firm. As mentioned previously, milestone financing allows additional flexibility in fashioning a contract. In the context of belief heterogeneity or venture capitalist liquidity preference, milestone financing allows tilting of the contract to respond to the above preferences of the contracting parties.

These effects may interact with one another. For example, the flexibility in milestone financing may be valuable because of incentive effects, and also because of differential preferences across states by the contracting parties. The induced tilt in the optimal ownership of contingent claims generated by belief heterogeneity runs counter to that generated by venture capitalist liquidity preference. Furthermore, their combined effect may either reinforce or counter the induced tilt generated by incentive effects.

³ For the effect of entrepreneurial optimism on the choice of debt maturity, see Landier and Thesmar (2005).

We compare the results of milestone and round financing in various scenarios. For example, when the role of venture capitalist effort is small, and the first stage of financing is relatively low, incentivizing the entrepreneur dominates the contractual relationship and round financing is preferred to milestone financing. When it is the role on entrepreneurial effort that is small, however, the need for contract flexibility dominates, and milestone financing is preferred to round financing. With homogenous expectations and no liquidity preference, flexibility is unimportant if technological success leaves unchanged the relative roles of entrepreneurial and venture capitalist efforts. For moderate financing levels, unless the role of entrepreneurial effort is quite small, round financing is preferred. Marginalizing the role of effort by both parties, either heterogeneous beliefs or venture capitalist liquidity preference makes milestone financing preferred to round financing.

The basic model is introduced in Section 2. A numerical example illustrates the difference between milestone and round financing. Both milestone financing and round financing are then modeled. Section 3 discusses the relative advantages of each contract type, and shows it through a number of cases. Section 4 extends the model to allow heterogeneity of beliefs and liquidity preference, and presents additional results about the relative advantages of the contracts. Section 5 concludes. Relevant proofs are contained in the Appendix.

2. The model

The real technology of the firm has two possible outcomes, "failure" and "success", determining two possible technology states, indexed by $i \in \{1, 2\}$ respectively. Both states may result in positive cash flow for the firm; technological "failure" of state 1 denotes lower expected cash flow relative to technological "success" of state 2.

The outcome of the technology is determined and observed after the firm spends I cash at time 0. If the technology is successful, the firm needs to spend an additional J cash at time 1 to achieve the higher expected cash flow associated with success. The firm generates a single cash

flow at time 2, whose expectation depends partially upon the outcome of the technology. The first I dollars of corporate spending can be interpreted as initial R&D or product development, while success early in the firm life requires additional corporate spending J for marketing or to take the project development to the next level.

After the initial I has been financed and spent by the firm, the outcome of the technology is publicly revealed. Both the entrepreneur and the venture capitalist then have the opportunity to expend personally costly effort to increase expected corporate cash flow. At time 1, conditional on the technology being successful, the firm raises and spends an additional J.⁴ At time 2, the firm generates a cash flow, which is distributed between the entrepreneur and venture capitalist as per previous agreement.

The role of the venture capitalist includes both intervening to fix matters as well as assisting the company to the next level. In practice, this can take many forms, such as coaching and mentoring management to achieve a better professionalism, stepping in to replace management, conducting an external search for a new management, helping management "learn the ropes" if they lack the necessary skills applicable to a start-up firm or the appropriate industry, providing guidance through operations on the board of directors, and networking in the financial community in preparation for future syndication or a fruitful exit (e.g., Lerner 1995 and Hellmann and Puri 2002). All of these require commitments of time, of which the venture capitalist has a limited amount available.

Under milestone financing, the entrepreneur and venture capitalist agree to a contract at

⁴ Technological success thus allows the firm to be taken "to the next level," investing additional funds to receive higher future revenue. For example, success could be interpreted as successful product development, which naturally leads to spending additional funds in marketing a product. Alternatively, success could be developing an innovative technology or process, which naturally leads to spending additional funds to develop commercial applications of the technology. Of course, "success" should imply that the incremental expected cash flow to the firm in state 2 should be greater than the incremental investment J.

time 0 wherein the venture capitalist immediately supplies I cash to the firm, receiving the right to a claim (fractional share) $f_1 \in [0, 1]$ of the cash flow of the firm. It is simultaneously agreed that if the milestone is met (the technology is a success, reaching state 2), the venture capitalist will supply an additional J cash to the firm at time 1, increasing her share of the firm to $f_2 \in [f_1, 1]$. Thus, if technology is a failure (the firm is in state 1), the venture capitalist receives a share f_1 , while the entrepreneur receives the residual share $(1 - f_1)$. If technology is a success (the firm is in state 2), the venture capitalist supplies an additional J cash and receives share f_2 while the entrepreneur receives share $(1 - f_2)$.⁵

Alternatively, under round financing, the entrepreneur and venture capitalist agree to a contract at time 0 (the first round) wherein the venture capitalist immediately supplies I cash to the firm, receiving the right to a share $F \in [0, 1]$ of the cash flow of the firm. If the technology subsequently is a success, necessitating additional J in financing at time 1, the entrepreneur and venture capitalist enter the second financing round. In that financing round, the venture capitalist (or another financier) will agree to supply an additional J cash to the firm, in return for a share determined by negotiations at that time, diluting both the entrepreneur and original venture capitalist shares.

The difference between milestone and round financing is the following. Under milestone financing, the firm gets a commitment and terms at time 0 for all needed financing: I at time 0, and J at time 1 (conditional on technological success). Under round financing, the firm gets a commitment and terms at time 0 only for immediate financing needs: I at time 0. If technological success occurs, the firm negotiates terms at time 1 for an additional J in financing.⁶ Thus, milestone financing requires commitment to financial terms (J in funding and the share it buys)

⁵ Since only the realizations of the cash flow and the technology are observable, these are the most general possible contracts with non-negative payouts.

⁶ We distinguish between stages of financing and rounds of financing as follows. Stages of financing refer to the points in time when additional cash is received by the firm. Rounds of financing refer to stages with accompanying negotiation of financing terms. Thus, when the real technology is successful, both milestone

before entrepreneurial effort and technology realization occur. In contrast, round financing requires negotiation of terms afterward.

The magnitude of the expected cash flow of the firm depends upon the success of the real technology, entrepreneurial effort and venture capitalist effort. The cash flow of the firm, generated at time 2, is either unity or zero. The probability of achieving a cash flow of unity in state i equals $(a_i + b_i + c_i)$, where the base probability $c_i \geq 0$ is augmented by the respective effort levels expended by the entrepreneur and venture capitalist, $a_i \geq 0$ and $b_i \geq 0$.

After the first stage of financing I is invested, and subsequent resolution and revelation of technological uncertainty occurs, the entrepreneur and venture capitalist have the opportunities to expend effort. In state i , the personal cost to the entrepreneur of expending effort a_i is $a_i^2/(2A_i)$, where $A_i > 0$, while the personal cost to the venture capitalist of expending effort b_i is $b_i^2/(2B_i)$, where $B_i > 0$. Effort affects the probabilities of the possible cash flow outcomes. Thus, an outsider cannot perfectly infer effort expended. In this spirit, it is assumed that, although efforts are observable, they are not verifiable, and therefore not contractible.⁷

At time 0, both entrepreneur and venture capitalist assign probabilities p_1 and p_2 to the two states (technological failure and success, respectively).⁸ All parties are risk-neutral. The venture capitalist acts competitively in pricing contracts. The discount rate is zero.

To ensure that technological success makes the firm better off, it is assumed that $A_2 \geq A_1$ and $B_2 \geq B_1$. To ensure that the second stage of financing is always worthwhile under a technological success, it is assumed that $c_2 \geq c_1 + J$. It is also instructive to limit the analysis to cases where it is worthwhile to finance the first round (as opposed to aborting the project at

and round financing will have two stages in our model, but only round financing will have a negotiation in each node of financing.

⁷ In particular, this precludes a third type of contract wherein the share price in the second stage is explicitly conditioned on entrepreneurial effort.

⁸ This will be relaxed in Section 4, where the effects of heterogeneous beliefs about technological success are considered.

inception). For it, the expected cost of financing the project to completion, $I + p_2J$, should not exceed an upper limit on the expected base cash flow plus an additional bound on the value derived from effort: $I + p_2J \leq c + B/2 + [\max(0, A - B - c)]^2/[2(2A - B)]$, where $A = p_1A_1 + p_2A_2$, $B = p_1B_1 + p_2B_2$, and $c = p_1c_1 + p_2c_2$ are the expected values of A_i , B_i , and c_i , respectively.⁹

The following is the timeline for events. At time 0, a financing contract is specified. This may be a milestone contract over both stages, or a round contract for the first stage of financing. Either way, the firm raises (and spends) I cash. Uncertainty about the technology state i is fully resolved and revealed. The entrepreneur and venture capitalist choose respective state-dependent effort levels a_i and b_i (observable, but not verifiable, and therefore not contractible). At time 1, if the technology is a success, the firm raises (and spends) J cash at the second financing stage. With milestone financing, these funds are raised under the already agreed terms. With round financing, the terms of the new round is agreed at this time. At time 2, the firm generates a cash flow (of either unity or zero), which is distributed between entrepreneur and venture capitalist based on the earlier contracting.

2.1. A numerical example

We first present a simple numerical example to illustrate the differences between round and milestone financing.¹⁰ It is important to note that round financing is not a special case of milestone financing. Milestone financing includes both a commitment to finance at the second stage, and an agreed price (conditional on technological success). Under round financing, financing at the second stage will occur at the competitive price, determined at that future date. In particular, an agreement by the venture capitalist to commit to second stage financing at a price to be determined later is not milestone financing, as it fails to include a price commitment.

⁹ This constraint will be adjusted for heterogeneous beliefs. See the proof of Proposition 7.

¹⁰ For simplicity in this example, the entrepreneur's effort level is taken as discrete rather than continuous. The example otherwise conforms to the model.

Milestone financing determines the financing terms before efforts are undertaken. Since efforts are not contractible, financing terms cannot depend upon the effort levels under milestone financing. In contrast, the second stage of round financing is agreed upon after effort levels are observed. Therefore, under round financing, the [second stage] financing terms can depend upon effort levels. Thus, the two types of financing are distinct. In the following example, the outcome available under round financing cannot be achieved under milestone financing, or vice versa.

In this example, the firm requires \$20 in external financing in first-stage financing. After spending this \$20, one of two equally likely technological outcomes, success or failure, is determined by nature. Under failure, the firm generates a zero cash flow. Under success, the firm generates a base expected cash flow of \$40. However, if the entrepreneur undertakes effort (personally costing him \$18), followed by the firm raising and spending an additional \$20 in external (second-stage) financing, then the firm's expected cash flow is \$100 rather than \$40. The additional financing without the effort increases the expected cash flow from \$40 to \$60.

Under round financing, the venture capitalist provides \$20 of financing in the first stage, and is compensated with a share F of the firm. If the entrepreneur undertakes the effort, and another \$20 is raised and spent at the second stage, firm value will be \$100. If there is a second-stage investor, she will need to be compensated with 20% of the firm. This dilutes the first-stage venture capitalist claim to a share $.8F$; since technological success has .5 probability, the fair first-stage venture capitalist claim satisfies $.5(.8F)(\$100) = \20 , or $F = 50\%$. Therefore, the entrepreneur's (pre-dilution) claim is $1 - F = 50\%$. Conditional on technological success, if the entrepreneur undertakes effort, his claim is an expected $(.8)(50\%)(\$100) = \40 , while without effort, his claim is an expected $(50\%)(\$40) = \20 . The \$20 difference is enough to induce the entrepreneur to undertake effort (costing \$18), and the first-best outcome is achieved.

Under milestone financing, the venture capital agrees to finance \$20 at the first stage and an additional \$20 if technological success occurs, an overall expected financing of \$30. She is

compensated with a share f of the firm when the technology is successful. If the entrepreneur is expected to exert effort, then $.5f(\$100) = \30 , so the venture capitalist would receive an $f = 60\%$ share of the firm, and the entrepreneur would receive $1 - f = 40\%$. Conditional on technological success, if the entrepreneur undertakes effort, his claim is worth $40\%(\$100) = \40 , while without effort, his claim is worth $40\%(\$60) = \24 . The $\$16$ difference is insufficient to induce him to take $\$18$ of effort. The entrepreneur will not undertake effort, and the firm will be worth only $\$60$ under technological success. Therefore, the required venture capitalist claim satisfies $.5f(\$60) = \30 , or $f = 100\%$ of the firm. This is the only feasible milestone financing contract; the first-best is not achieved. Here, only round financing can achieve the first-best outcome; this cannot be replicated by any milestone financing contract.

Changing the example slightly can make milestone financing the optimal contract type. Suppose that the only change is that, under technological failure, the entrepreneur can increase the expected firm value by $\$15$ if he exerts $\$10$ of effort. Note that the milestone contract giving the venture capitalist 55% of the firm under technological success and $33\frac{1}{3}\%$ of the firm under technological failure generates first-best incentives in both states: it is worthwhile for the entrepreneur to exert effort in the first case since $.45[.8(\$100) - \$40] = \$18$, and in the second because $(2/3)(\$15 - \$0) = \$10$. Such a contract is feasible since $.5(.55)\$100 + .5(1/3)\$15 = \$30$, the expected milestone financing. However, no round financing can generate the first-best: to generate effort under technological failure requires the venture capitalist share $F \leq 33\frac{1}{3}\%$, which is not feasible because $.5(1/3)\$80 + .5(1/3)\$15 < \$20$, the first-stage round financing. Thus, only a milestone contract can achieve the first-best effort.

2.2. Milestone financing

We now turn to the analysis of the general model. Determining the optimal milestone financing contract requires backward induction. Venture capitalist and entrepreneurial efforts must be determined based on the particular milestone financing contract characterized by the shares (f_1, f_2) .

In determining her optimal effort level, the venture capitalist maximizes her share of expected corporate cash flow less her personal cost of effort,

$$\begin{aligned} \text{Max} \quad & f_i (a_i + b_i + c_i) - b_i^2/(2B_i), \\ & b_i \geq 0 \end{aligned}$$

implying optimal venture capitalist effort $b_i^* = B_i f_i$. Similarly, in determining his optimal level of effort, the entrepreneur maximizes his share of expected corporate cash flow less his personal cost of effort,

$$\begin{aligned} \text{Max} \quad & (1 - f_i) (a_i + b_i + c_i) - a_i^2/(2A_i), \\ & a_i \geq 0 \end{aligned}$$

implying optimal entrepreneurial effort $a_i^* = A_i (1 - f_i)$. Note that the efforts levels are independent of the order in which the parties undertake their efforts.

A feasible milestone contract must offer the venture capitalist a claim on the firm that is large enough to reimburse her for providing funds, while recognizing her cost of undertaking effort. The milestone contract is a commitment for the venture capitalist to provide I at the first stage, and J at the second stage (conditional on technological success), for an expected $I + p_2 J$ in total capital provided. Feasible contracting requires

$$\begin{aligned} I + p_2 J &= \sum_i p_i [f_i (a_i^* + b_i^* + c_i) - (b_i^*)^2/(2B_i)] \\ &= \sum_i p_i [(A_i + c_i) f_i - (A_i - B_i/2) f_i^2], \end{aligned} \tag{1}$$

after substituting for a_i^* and b_i^* .¹¹

The optimal milestone contract maximizes the value of the entrepreneur's residual claim at time 0, less the cost of entrepreneurial effort. Define Φ_M as the entrepreneur's objective function under milestone financing:

$$\begin{aligned}\Phi_M &= \sum_i p_i [(1 - f_i) (a_i^* + b_i^* + c_i) - (a_i^*)^2/(2A_i)] \\ &= \sum_i p_i [(a_i^* + b_i^* + c_i) - (a_i^*)^2/(2A_i) - (b_i^*)^2/(2B_i)] - (I + p_2J) \\ &= \sum_i p_i [(A_i/2)(1 - f_i^2) + (B_i/2)(2f_i - f_i^2) + c_i] - (I + p_2J),\end{aligned}$$

using (1) and substituting for a_i^* and b_i^* . The optimal milestone contract thus solves

$$\begin{aligned}\text{Max } \Phi_M &= \sum_i p_i [(A_i/2)(1 - f_i^2) + (B_i/2)(2f_i - f_i^2) + c_i] - (I + p_2J), \\ 0 \leq f_1 \leq f_2 \leq 1\end{aligned}\tag{2}$$

$$\text{subject to: } I + p_2J = \sum_i p_i [(A_i + c_i) f_i - (A_i - B_i/2) f_i^2].$$

2.3. Round financing

Determining the round financing contract proceeds as follows. At the first stage of financing, the venture capitalist receives a share F of the firm. If technology state 2 occurs, there is a second stage of financing, where the venture capitalist receives a new share G of the firm as compensation for capital of J . As this is an "up" round of financing, both the entrepreneur's share $(1 - F)$ and previous venture capitalist share F of the firm are diluted by a factor $(1 - G)$. The competitive price for the new financing satisfies $G(a_2 + b_2 + c_2) = J$.

Venture capitalist effort in state 1 is determined by:

¹¹ We assume that the claim the venture capitalist receives exactly compensates her for providing funds and her expected effort.

$$\begin{aligned} \text{Max} \quad & F(a_1 + b_1 + c_1) - b_1^2/(2B_1), \\ & b_1 \geq 0 \end{aligned}$$

implying $b_1^* = B_1F$. Venture capitalist effort in state 2 recognizes the dilution, and is determined by:

$$\begin{aligned} \text{Max} \quad & F(1 - G)(a_2 + b_2 + c_2) - b_2^2/(2B_2) = F(a_2 + b_2 + c_2) - FJ - b_2^2/(2B_2), \\ & b_2 \geq 0 \end{aligned}$$

implying that $b_2^* = B_2F$.

Similarly, entrepreneurial effort in state 1 is determined by:

$$\begin{aligned} \text{Max} \quad & (1 - F)(a_1 + b_1 + c_1) - a_1^2/(2A_1), \\ & a_1 \geq 0 \end{aligned}$$

implying $a_1^* = A_1(1 - F)$. Entrepreneurial effort in state 2 is determined by

$$\begin{aligned} \text{Max} \quad & (1 - F)(1 - G)(a_2 + b_2 + c_2) - a_2^2/(2A_2) = (1 - F)(a_2 + b_2 + c_2) - (1 - F)J - a_2^2/(2A_2), \\ & b_2 \geq 0 \end{aligned}$$

implying $a_2^* = A_2(1 - F)$. Again, these results are independent of the order in which effort is undertaken.

Feasible contracting for the venture capitalist requires

$$I = \sum_i p_i [F(a_i^* + b_i^* + c_i) - (b_i^*)^2/(2B_i)] - p_2JF,$$

or

$$I + p_2JF = \sum_i p_i [(A_i + c_i)F - (A_i - B_i/2)F^2]. \quad (3)$$

Under round financing, the residual value to the entrepreneur at time 0, Φ_R , can be written as

$$\begin{aligned}
\Phi_R &= \sum_i p_i [(1 - F)(a_i^* + b_i^* + c_i) - (a_i^*)^2/(2A_i)] - p_2 J(1 - F) \\
&= \sum_i p_i [(a_i^* + b_i^* + c_i) - (a_i^*)^2/(2A_i) - (b_i^*)^2/(2B_i)] - (I + p_2 J) \\
&= \sum_i p_i [(A_i/2)(1 - F^2) + (B_i/2)(2F - F^2) + c_i] - (I + p_2 J),
\end{aligned}$$

using (3) and substituting for a_i^* and b_i^* . The round financing contract is therefore specified by

$$\Phi_R = \sum_i p_i [(A_i/2)(1 - F^2) + (B_i/2)(2F - F^2) + c_i] - (I + p_2 J), \quad (4)$$

$$\text{subject to: } I + p_2 J F = \sum_i p_i [(A_i + c_i) F - (A_i - B_i/2) F^2].$$

3. Comparing milestone and round financing

The primary intuition for our results is as follows. The advantage associated with milestone financing is contract flexibility.¹² With round financing, funds are raised at each stage based upon the fair value of the firm at that point. However, with milestone financing, a contract can be written assigning claims to each party over multiple possible future outcomes (technological success or failure). These claims need not be designed so that they are *ex post* priced fairly in each possible outcome; they need only be designed so that they are *ex ante* priced fairly, before knowing the outcome. This gives additional flexibility to milestone contracting relative to round financing. Thus, a milestone contract could be written to give one party a relatively disproportionate claim in one state, as long as the other party is appropriately compensated by adjusting their claim in the other state.

The advantage associated with round financing is increased entrepreneurial incentive. Because round financing has a lower upfront commitment than milestone financing, consisting

¹² Somewhat paradoxically, because milestone financing involves earlier commitment, it offers more flexibility than round financing.

only of a commitment to finance the current stage, the venture capitalist receives lower compensation upfront. Therefore, the entrepreneur captures more of any increase in the firm value between the first and second financing stages, resulting in an increased incentive to expend personally costly effort. Of course, since the venture capitalist has a reduced incentive to take costly effort, this is in advantage only when entrepreneurial effort is relatively more important than venture capitalist effort; if venture capitalist effort was more important, the advantage would reverse. The round financing advantage arises because round financing gives the venture capitalist a relatively smaller fraction of the firm between financing stages; we will therefore refer to this as incentive shifting.

For a particular firm, the preferred method of contracting, whether milestone financing or round financing, is determined by whether the advantage associated with milestone financing (flexibility) outweighs the advantage associated with round financing (the increased entrepreneurial incentive). To illustrate the cases when the entrepreneurial incentive effect outweighs the flexibility effect, and conversely, we consider a number of special parametric cases.

We first consider the case where venture capitalist effort is unimportant relative to entrepreneurial effort, and the financing required in the first stage is small. In the extreme, $B_1 = B_2 = 0$ and $I = 0$. In this extreme case, round financing gives the venture capitalist no claim at the first stage, since no commitment is made. Milestone financing gives the venture capitalist a positive claim at the first stage, as long as $p_2J > 0$, as compensation for a future cash commitment. Since only entrepreneurial incentives matter, round financing achieves the first best effort, which milestone does not. Therefore, round financing is preferred to milestone financing here. (All proofs are in the Appendix.)

Proposition 1. *When venture capitalist effort is relatively unimportant and the first stage financing requirement is relatively small (B_1 , B_2 , and I small), round financing is preferred to milestone financing.*

Although we believe it less likely to occur in practice, we next consider the case where entrepreneurial effort is considerably less important relative to venture capitalist effort, and again the financing required in the first stage is small. In the extreme, $A_1 = A_2 = 0$ and $I = 0$. In this case, because round financing gives the venture capitalist no claim at the first stage, it generates no incentives for the venture capitalist to undertake effort. In contrast, milestone financing gives the venture capitalist some incentive to undertake effort. Since only venture capitalist effort matters, milestone financing is preferred to round financing here.

Proposition 2. *When entrepreneurial effort is relatively unimportant and the first stage financing requirement is relatively small (A_1 , A_2 , and I small), milestone financing is preferred to round financing.*

We next consider the case where the expected financing requirement in the second stage is relatively small. This could be either because the magnitude of the second stage financing is small, or because reaching the second stage (technological success) is relatively unlikely. In the extreme case, $p_2J = 0$. The incentive shifting effect disappears at this extreme, as milestone and round financing require the same expected dollar commitment by the venture capitalist. Since only the flexibility effect remains, milestone financing is generally preferred to round financing. (With a few specific parameter values, they may be equally good.)

Proposition 3. *When either the probability of technological success is relatively small, or the second stage financing requirement is relatively small (p_2J small), milestone financing is preferred to round financing.*

Next, we consider the interesting case where the effect of technological success is to scale up the firm's production technology. Specifically, the parameters A_i , B_i , and c_i are proportional across technological outcomes: $A_2 = \mu A_1$, $B_2 = \mu B_1$, and $c_2 = \mu c_1$, with $\mu \geq 1$. Under this condition, the

optimal milestone contract requires a very high implied price of equity at the second stage,¹³ so that the venture capitalist claims are the same across technological outcomes, $f_2 = f_1$. Therefore, the flexibility of milestone financing (being able to assign disparate sized claims across technological states) has no value. Only the incentive shifting effect remains. The venture capitalist has a smaller claim, and the entrepreneur has a larger claim between the two financing dates under round financing. As long as entrepreneurial effort is more important than venture capitalist effort, and both stages of financing are relatively meaningful, this results in round financing being preferred. Sufficient conditions are: $I \geq [c - p_2J + (2A^2 + B^2)/2(A + B)]B/(A + B)$, and $p_2J > 0$, where $A = p_1A_1 + p_2A_2$, $B = p_1B_1 + p_2B_2$, and $c = p_1c_1 + p_2c_2$ are the expected values of A_i , B_i , and c_i , respectively.

Proposition 4. *When technological success scales the firm's production technology, $I \geq [c - p_2J + (2A^2 + B^2)/2(A + B)]B/(A + B)$, and $p_2J > 0$, round financing is preferred to milestone financing.*

3.1 Renegotiation

It is conceivable that the entrepreneur and venture capitalist will wish to renegotiate their contract, after the technological state is revealed, in order to change the incentives to provide effort. Whether milestone or round financing, the contract is initially agreed before the technology outcome is known, so it is based upon the expected effort incentives generated across the two states. After the technological outcome is known, the (state conditional) effort incentives are typically different than the (unconditional) average incentives across the states. At an extreme, it is possible that the entrepreneur or venture capitalist would even freely give up part of their corporate claim to the other party if that lead to sufficient improvement in the incentives to provide effort.

¹³ Although this milestone contract requires very high equity prices, milestone contracting is not optimal.

If the only contracts considered are those which are renegotiation proof, our results do not change significantly. Propositions 1, 2, and 3 remain unchanged, while Proposition 4 will still hold subject to an additional parametric constraint that entrepreneurial effort is not overwhelmingly costly.

***Proposition 5.** Suppose only renegotiation proof contracts are allowed. Then Propositions 1, 2, and 3 still hold. Proposition 4 holds with the additional constraint $A_2 \leq B_2 + c_2 - J$.*

4. Heterogeneous beliefs and liquidity preference

This section extends the previous model by considering the situation where the entrepreneur and venture capitalist place different values on the same contingent claims. There are two ways in which we allow this to occur. The entrepreneur and venture capitalist may have different expectations (at time 0) of the probability for technological success. One might naturally expect that a venture capitalist willing to enter into a partnership with the entrepreneur to be optimistic about the probability of success, relative to other potential venture capitalists who elected not to become involved with the project. However, the entrepreneur is likely to be more optimistic still.¹⁴ Let p_1 and p_2 be the probability that the entrepreneur assigns to technological failure and success; let q_1 and q_2 be the probability that the venture capitalist assigns to technological failure and success, with $p_2 \geq q_2$. Hence, at time 0, the entrepreneur and venture capitalist may be willing to pay different amounts for a claim of one dollar contingent on state i occurrence.

Another possibility is recognizing the venture capitalist's preference for liquidity. As previously noted, venture capitalists raise funds through limited partnerships with a finite life. This generates a strong preference for harvesting every investment before the end of the fund life.

¹⁴ There is overwhelming evidence that entrepreneurs tend to overestimate their chances of success. See Landier and Thesmar (2005) for recent evidence and for further references.

The venture capitalist may thus be hesitant to commit funds to a long-lived enterprise.¹⁵ However, if the corporate technology is successful, it is more likely that the firm may become more liquid (for example, the firm is more likely to be floated through an initial public offering, or the firm reaches a critical mass that triggers a take over by another company for cash or publicly traded equity). We model this by having the venture capitalist value one dollar of [illiquid] assets of the firm in state 1 at value $L \leq 1$, while fully valuing one dollar of [liquid] assets in state 2.

To some extent, the effects of heterogeneous beliefs and liquidity preference are offsetting. Consider a pair of contingent claims, each paying off one dollar in state i . Relative to the entrepreneur's valuation, liquidity preference tends to make the venture capitalist prefer claims in state 2 (valued for being more liquid), while heterogeneity tends to make the venture capitalist prefer claims in state 1 (the entrepreneur is even more optimistic than the venture capitalist about the likelihood of technological success).

To clarify, suppose there is neither the opportunity for entrepreneurial effort nor venture capitalist effort ($A_i = B_i = 0$), but heterogeneous beliefs and venture capitalist liquidity preference are allowed. Under milestone financing, the venture capitalist feasibility constraint is

$$I + q_2J = Lq_1f_1c_1 + q_2f_2c_2, \quad (5)$$

and, using equation (5), the entrepreneur's objective is

$$\begin{aligned} \text{Max } \Phi_M &= p_1(1 - f_1)c_1 + p_2(1 - f_2)c_2 = c + (1 - p_2/q_2)I + Zp_1c_1f_1, \\ 0 &\leq f_1 \leq f_2 \leq 1 \\ \text{subject to: } & I + q_2J = Lq_1c_1f_1 + q_2c_2f_2, \end{aligned} \quad (6)$$

¹⁵ Thus, liquidity preference is generated by the venture capitalist having a shorter investment horizon than the entrepreneur.

where $Z = (Lp_2q_1/p_1q_2 - 1)$ captures heterogeneity and liquidity preference. The parameter Z is increasing in belief heterogeneity (increasing p_2 or decreasing q_2) and decreasing in liquidity preference (decreasing L).

Under round financing, the entrepreneur's expected value is

$$\Phi_R = p_1(1 - F)c_1 + p_2(1 - F)(c_2 - J) = c + (1 - p_2/q_2)I + Zp_1c_1F, \quad (7)$$

$$\text{subject to: } I + q_2JF = Lq_1c_1F + q_2c_2F.$$

Thus, the entrepreneur's objective function under milestone and round financing are essentially the same, except for the last term (with Z), which depends upon f_1 or F , for milestone and rounds, respectively. Heterogeneity and liquidity effects are captured in the Z variable. With homogeneity and no liquidity preference ($p_2 = q_2$, $L = 1$), Z is zero; neither milestone nor round financing offers a relative advantage. With heterogeneity only ($p_2 > q_2$, $L = 1$), Z is positive; with liquidity preference only ($p_2 = q_2$, $L < 1$), Z is negative; with both heterogeneity and liquidity preference ($p_2 > q_2$, $L < 1$), Z can be positive, negative, or even zero. In the absence of effort, the round financing outcome can be replicated by milestone financing. But in general, the round financing outcome cannot be replicated by milestone financing. Since flexibility is generally valuable, milestone financing is preferred to round financing (with indifference over contract types when $Z = 0$).

Proposition 6. *Suppose neither entrepreneurial nor venture capitalist effort is important (A_i and B_i small), but heterogeneous beliefs about the probability of technological success or venture capitalist liquidity preference are allowed. Then milestone financing is generally preferred to round financing.*

We next consider the effects of heterogeneity and liquidity preference combined with entrepreneurial and venture capitalist effort. In two previously examined cases (Propositions 2 and

3), the incentive shifting effect disappears, and only the flexibility effect remains. As flexibility is generally valuable with heterogeneous beliefs or liquidity preference, introducing heterogeneity or liquidity preference to these two cases will still result in a preference for milestone financing. Note that the introduction of heterogeneous beliefs or liquidity preference need not increase the preference for flexibility. For example, without heterogeneity/liquidity, there may be a preference to increase f_2 relative to f_1 , while introducing heterogeneity/liquidity generates a preference to decrease f_2 relative to f_1 . The effects may partially offset; nevertheless, the ability to be flexible remains valuable.

Proposition 7. *Suppose heterogeneous beliefs about the probability of technological success or venture capitalist liquidity preference are allowed. Then in the following cases, milestone financing remains generally preferred to round financing.*

- *When entrepreneurial effort is relatively unimportant and the first stage financing requirement is relatively small (A_1 , A_2 , and I small)*
- *When either the probability of technological success is relatively small, or the second stage financing requirement is relatively small ($q_2 J$ small)*

Both heterogeneity and liquidity preference generate tendencies to change the mix of claims under milestone financing. Heterogeneity tends to tilt the mix toward a smaller f_2 , as the venture capitalist is relatively less optimistic than the entrepreneur about the likelihood of achieving state 2 (technological success). Liquidity preference tends to tilt the mix toward a larger f_2 , as the venture capitalist particularly values the liquidity of state 2, and furthermore has decreased incentive to work in state 1 to achieve illiquid cash flows. Heterogeneity and liquidity preference also change the magnitude of the claims under both milestone and round financing, both because of the changing relative valuation of contingent claims across the states by the two contracting parties, as well as the changed incentive for the venture capitalist to exert effort with

liquidity preference. Thus, the presence of heterogeneity and liquidity preference leads to complex, and often offsetting effects.

Since the two contracting parties value state-contingent claims differently, the ability to exchange state-contingent claims can potentially create value. In the presence of heterogeneity, a larger upfront commitment from the venture capitalist allows more claims to pass into her hands at the first financing stage; there is an opportunity to create value by giving her more of the claims she values relatively highly. This works in the opposite direction to the incentive shifting effect (in which a smaller upfront commitment improves entrepreneurial incentives). With liquidity preference, a larger upfront commitment from the venture capitalist allows more claims to pass into her hands; since her value of the claims is (weakly) less than that of the entrepreneur, this destroys value. This works in the same direction as the incentive shifting effect. Thus, if liquidity preference dominates heterogeneity, the previously examined case (Proposition 1) of preference for round financing is only reinforced by the addition of liquidity preference.

***Proposition 8.** Suppose venture capitalist liquidity preference is at least as important as heterogeneity ($Z \leq 0$). When venture capitalist effort is relatively unimportant and the first stage financing requirement is relatively small (B_1 , B_2 , and I small), round financing remains preferred to milestone financing.*

5. Conclusion

The two commonly used methods of implementing staged financing for start-up firms are milestone financing, in which the firm receives a commitment for additional injections of financing after certain criteria (milestones) have been reached; and round financing, in which the firm has no explicit commitment, but goes to the venture capital market for additional financing (where it presumably can receive financing if it shows sufficient progress.) This paper has examined the difference in the two contracting methods, concentrating on the effect the contracts

can have on the incentives of the contracting parties. We assume both of the contracting parties, entrepreneur and venture capitalist, are able to undertake personally costly effort which has positive effects on the cash flow of the firm. Although both of these can affect enterprise cash flow, the way in which they can do so differs. The entrepreneur, intimately involved with the day-to-day operation of the firm, is assumed to be able to directly affect cash flows with hard work in the early stages of the firm. The venture capitalist has more of a strategic role. In times of crisis, she is able to step in to guide management, help replace management, or offer strategic thinking.

In considering an optimal contract type, the primary effects we examine are the effects of the contract on the entrepreneur and venture capitalist's incentives. We also examine the effect of differential expectations about the likelihood of success on the real technology and on the venture capitalist's preference for liquid investments on the optimal contract. We work in a context of fully symmetric information and risk-neutral parties. This is a rich context, allowing complex effects. The preferred contract type depends upon the relative amount of needed financing across the two stages, the relative abilities of the parties to affect the outcome, the probability of technological success, heterogeneity of beliefs, strength of liquidity preference, and the extent to which technological success scales up corporate cash flows.

We are able to characterize various sets of parameters for which milestone financing is preferred to round financing, and other sets of parameters for which round financing is preferred to milestone financing. For example, when the role of the entrepreneur is much more important than that of the venture capitalist in affecting firm fortunes, and the financing required in the first stage is relatively low, round financing is more effective. When instead venture capitalist effort is much more important, milestone financing is more effective. When the real technology is a long-shot, milestone financing is more effective. When technological success results in a simple scaling of cash flows and sensitivities to effort, round financing is more effective (unless the role of the venture capitalist is much more important than the entrepreneur). When no one's effort is

important, the presence of either belief heterogeneity or venture capitalist liquidity preference implies milestone financing is preferable. Thus, the nature of the enterprise, its cash flows, and sensitivity of its cash flows to entrepreneurial effort, venture capitalist effort, and success of the underlying real technology are important determinants of which of milestone financing and round financing is preferred.

Appendix

Proof of Proposition 1. When venture capitalist effort is relatively unimportant and the first stage financing requirement is relatively small (B_1 , B_2 , and I small), round financing is preferred to milestone financing.

First, note that both milestone and round financing contracts are feasible. Let $B_1 = B_2 = 0$ and $I =$

0. The optimal milestone contract (2) reduces to

$$\begin{aligned} \text{Max } \Phi_M &= \sum_i p_i [(A_i/2)(1 - f_i^2) + c_i] - p_2 J, \\ 0 \leq f_1 \leq f_2 \leq 1 \\ \text{subject to: } \quad p_2 J &= \sum_i p_i [(A_i + c_i) f_i - A_i f_i^2], \end{aligned} \quad (\text{A1})$$

and the round financing contract (4) reduces to

$$\begin{aligned} \Phi_R &= \sum_i p_i [(A_i/2)(1 - F^2) + c_i] - p_2 J, \\ \text{subject to: } \quad p_2 J F &= \sum_i p_i [(A_i + c_i) F - A_i F^2]. \end{aligned} \quad (\text{A2})$$

The round financing contract has $F = 0$. Since Φ_R is decreasing in F , this is first best. From the constraint in (A1), f_1 and f_2 are non-negative. If $p_2 J > 0$, at least one must be positive, and no feasible milestone financing contract can achieve first best. ♣

Proof of Proposition 2. When entrepreneurial effort is relatively unimportant and the first stage financing requirement is relatively small (A_1 , A_2 , and I small), milestone financing is preferred to round financing.

Let $A_1 = A_2 = 0$ and $I = 0$. The optimal milestone contract (2) reduces to

$$\begin{aligned} \text{Max } \Phi_M &= \sum_i p_i [(B_i/2)(2f_i - f_i^2) + c_i] - p_2 J, \\ 0 \leq f_1 \leq f_2 \leq 1 \\ \text{subject to: } \quad p_2 J &= \sum_i p_i [c_i f_i + (B_i/2) f_i^2], \end{aligned} \quad (\text{A3})$$

and the round financing contract (4) reduces to

$$\begin{aligned} \Phi_R &= \sum_i p_i [(B_i/2)(2F - F^2) + c_i] - p_2 J, \\ \text{subject to: } \quad p_2 J F &= \sum_i p_i [c_i F + (B_i/2) F^2]. \end{aligned} \quad (\text{A4})$$

The round financing contract has $F = 0$. Therefore, $\Phi_R = \sum_i p_i c_i - p_2 J$. From the constraint in (A3), f_1 and f_2 are non-negative. If $p_2 J > 0$, at least one must be positive, and $\Phi_M - \Phi_R = \sum_i p_i [(B_i/2)(2f_i - f_i^2)] > 0$. ♣

Proof of Proposition 3. When either the probability of technological success is relatively small, or the second stage financing requirement is relatively small ($p_2 J$ small), milestone financing is preferred to round financing.

Let $p_2 J = 0$. The optimal milestone contract (2) reduces to

$$\begin{aligned} \text{Max } \Phi_M &= \sum_i p_i [(A_i/2)(1 - f_i^2) + (B_i/2)(2f_i - f_i^2) + c_i] - I, \\ 0 &\leq f_1 \leq f_2 \leq 1 \\ \text{subject to: } I &= \sum_i p_i [(A_i + c_i) f_i - (A_i - B_i/2) f_i^2], \end{aligned} \quad (\text{A5})$$

and the round financing contract (4) reduces to

$$\begin{aligned} \Phi_R &= \sum_i p_i [(A_i/2)(1 - F^2) + (B_i/2)(2F - F^2) + c_i] - I, \\ \text{subject to: } I &= \sum_i p_i [(A_i + c_i) F - (A_i - B_i/2) F^2]. \end{aligned} \quad (\text{A6})$$

Note that $f_1 = f_2 = F$ is a feasible milestone contract. Here, a milestone contract can replicate the round contract, while the optimal milestone contract need not have $f_1 = f_2$, so the optimal milestone contract is at least as good, and if $f_1 \neq f_2$, is better than the round contract. ♣

Proof of Proposition 4. When technological success scales the firm's production technology, $I \geq [c - p_2 J + (2A^2 + B^2)/2(A + B)]B/(A + B)$, and $p_2 J > 0$, round financing is preferred to milestone financing.

Let $A_2 = \mu A_1$, $B_2 = \mu B_1$, $c_2 = \mu c_1$, $p_2 J > 0$, and $I \geq [c - p_2 J + (2A^2 + B^2)/2(A + B)]B/(A + B)$. To solve for the optimal milestone contract in (2), use Lagrange multipliers. The first-order condition is either $f_1 < f_2$ and $\lambda(f_1) = \lambda(f_2)$, or $f_1 = f_2$ and $\lambda(f_1) \geq \lambda(f_2)$, where λ is defined by

$$\lambda(f) = [B - (A + B)f]/[A + c - (2A - B)f]. \quad (\text{A7})$$

If $f_1 \neq f_2$, then $\lambda(f_1) \neq \lambda(f_2)$ unless λ is constant. Therefore, without loss of generality, the optimal milestone contract can be taken to have $f_1 = f_2$. The optimal milestone contract (2) reduces to

$$\Phi_M = (A/2) + c - (I + p_2J) + Bf_1 - (A + B)f_1^2/2, \quad (A8)$$

$$\text{subject to: } I + p_2J = (A + c) f_1 - (A - B/2) f_1^2,$$

and the round financing contract (4) can be written

$$\Phi_R = (A/2) + c - (I + p_2J) + BF - (A + B)F^2/2, \quad (A9)$$

$$\text{subject to: } I + p_2JF = (A + c) F - (A - B/2) F^2.$$

It follows that $f_1 > F$. Solving the quadratic constraint in (A9),

$$F = \left((A + c - p_2J) - [(A + c - p_2J)^2 - 2I(2A - B)]^{1/2} \right) / (2A - B) \geq B/(A + B)$$

exactly when $I \leq [c - p_2J + (2A^2 + B^2)/2(A + B)]B/(A + B)$. Note that Φ_R is decreasing above $B/(A + B)$. It follows that round financing is preferred. ♣

Proof of Proposition 5. Suppose only renegotiation proof contracts are allowed. Then Propositions 1, 2, and 3 still hold. Proposition 4 holds with the additional constraint $A_2 \leq B_2 + c_2 - J$.

Without an additional influx of capital, the only possible renegotiation involves either the entrepreneur or venture capitalist voluntarily giving up part of their claim to the other party after the technological outcome is known, but before efforts are undertaken, in order to improve the other party's incentive to undertake effort.

Under milestone financing, just after state i has been revealed, the value of the venture capitalist claim (net of effort) is

$$VC_M = (A_i + c_i) f_i + (B_i/2 - A_i) f_i^2,$$

and the value of the entrepreneurial claim (net of effort) is

$$EN_M = c_i + A_i/2 + (B_i - A_i - c_i)f_i + (A_i/2 - B_i) f_i^2.$$

Under round financing, the corresponding values are

$$VC_R = (A_i + c_i - j_i) F + (B_i/2 - A_i) F^2,$$

$$EN_R = c_i - j_i + A_i/2 + (B_i - A_i - c_i + j_i)F + (A_i/2 - B_i) F^2,$$

where $j_1 = 0$ and $j_2 = J$ are the remaining uncommitted second stage financing needs. Note $c_i \geq j_i$.

For Proposition 1, when $B_i = I = 0$, under round financing, $F = 0$; the venture capitalist has no claim to give up. Since $\partial EN_R / \partial F = -A_i(1 - F) - (c_i - j_i) \leq 0$, the entrepreneur will not give up part of his claim. No renegotiation happens under the optimal (round) contract. For Proposition 2, when $A_i = I = 0$, under milestone financing, $\partial VC_M / \partial f_i = c_i + B_i f_i \geq 0$, so the venture capitalist will not give up part of her claim. Since $\partial EN_M / \partial f_i = (B_i - c_i) - 2B_i f_i$, the entrepreneur will not give up part of his claim unless $f_i \leq (B_i - c_i) / 2B_i$, and $B_i - c_i \geq 0$. However, in that case, under round financing, $F = 0$, and $\partial EN_R / \partial F = (B_i - c_i + j_i) - 2B_i F$, so the round financing contract will be renegotiated as well (with even greater potential gains from renegotiation than the optimal milestone contract). For Proposition 3, compare feasibility constraints (1) and (3). When $p_2 J = 0$, every feasible round contract F is replicable by a milestone contract $f_1 = f_2 = F$. For that milestone contract, the claims VC_M and EN_M are identical to VC_R and EN_R . Therefore, all renegotiation proof round contracts are replicable by renegotiation proof milestone contracts. For Proposition 4, if $A \geq 2B$, then $\partial EN_R / \partial F \leq (B_i - A_i - c_i + j_i) + (A_i - 2B_i) \leq 0$. If $A < 2B$, since $F \geq B / (A + B)$, then $\partial EN_R / \partial F \leq -(c_i - j_i) - (A_i^2 - A_i B_i + B_i^2) / (A_i + B_i) \leq 0$; the entrepreneur will not give up part of her claim. Since $A_2 \leq B_2 + c_2 - J$, then $A_1 \leq B_1 + c_1$, and $A_i \leq B_i + c_i - j_i$. For $B \geq 2A$, $\partial VC_R / \partial F \geq A_i + c_i - j_i \geq 0$, while for $B < 2A$, $\partial VC_R / \partial F \geq A_i + c_i - j_i + (B_i - 2A_i) = -A_i + B_i + c_i - j_i \geq 0$; the venture capitalist will not give up part of his claim, so no renegotiation of the optimal (round) contract occurs. ♣

Proof of Proposition 6. Suppose neither entrepreneurial nor venture capitalist effort is important (A_i and B_i small), but heterogeneous beliefs about the probability of technological success or venture capitalist liquidity preference are allowed. Then milestone financing is generally preferred to round financing.

Note that round financing can be replicated by milestone financing with $f_2 = F + (1 - F)J/c_2$ and $f_1 = F$. Compare the objectives in (6) and (7). If $Z = 0$, then both contract types do equally well. If $Z > 0$, then milestone financing can improve on round financing by increasing f_1 (and decreasing f_2). If $Z < 0$, milestone financing can improve on round financing by decreasing f_1 (and increasing f_2). ♣

Proof of Proposition 7. Suppose heterogeneous beliefs about the probability of technological success or venture capitalist liquidity preference are allowed. Then in the following cases, milestone financing remains generally preferred to round financing.

- When entrepreneurial effort is relatively unimportant and the first stage financing requirement is relatively small (A_1, A_2 , and I small)
- When either the probability of technological success is relatively small, or the second stage financing requirement is relatively small (q_2J small)

We first develop the general milestone and round financing problems under heterogeneity and liquidity preference. Let $L_1 = L$ and $L_2 = 1$. Under milestone financing, similar to the development in Section 2 of the paper, the optimal venture capitalist and entrepreneurial effort levels are $b_i^* = B_i L_i f_i$ and $a_i^* = A_i(1 - f_i)$. Feasible contracting requires

$$\begin{aligned} I + q_2J &= \sum_i q_i [f_i L_i (a_i^* + b_i^* + c_i) - (b_i^*)^2 / (2B_i)] \\ &= \sum_i q_i [(A_i + c_i) L_i f_i - (A_i L_i - B_i L_i^2 / 2) f_i^2]. \end{aligned} \quad (A10)$$

The entrepreneur's objective function $\Phi_M = \sum_i p_i [(1 - f_i) (a_i^* + b_i^* + c_i) - (a_i^*)^2 / (2A_i)]$. Substituting from (A10), the optimal milestone contract solves

$$\begin{aligned} \text{Max } \Phi_M &= \sum_i p_i [(A_i/2)(1 - f_i^2) + (B_i L_i/2)(2f_i - f_i^2) + c_i] - (p_2/q_2)I - p_2J \\ 0 \leq f_1 \leq f_2 \leq 1 & \quad + Z p_1 f_1 [A_1(1 - f_1) + B_1 L_1 f_1/2 + c_1], \quad (A11) \\ \text{subject to: } I + q_2J &= \sum_i q_i [(A_i + c_i) L_i f_i - (A_i L_i - B_i L_i^2/2) f_i^2]. \end{aligned}$$

Under round financing, the optimal venture capitalist and entrepreneurial effort levels are $b_i^* = B_i L_i F$ and $a_i^* = A_i (1 - F)$. Feasible contracting requires:

$$\begin{aligned} I &= \sum_i q_i [F L_i (a_i^* + b_i^* + c_i) - (b_i^*)^2 / (2B_i)] - q_2 J F \\ &= \sum_i q_i [(A_i + c_i) L_i F - (A_i L_i - B_i L_i^2 / 2) F^2] - q_2 J F. \end{aligned} \quad (A12)$$

The entrepreneur's objective function $\Phi_R = \sum_i p_i [(1 - F) (a_i^* + b_i^* + c_i) - (a_i^*)^2 / (2A_i)]$. Substituting from (A12), the round contract solves

$$\begin{aligned} \Phi_R &= \sum_i p_i [(A_i / 2)(1 - F^2) + (B_i L_i / 2)(2F - F^2) + c_i] - (p_2 / q_2) I - p_2 J \\ &\quad + Z p_1 F [A_1 (1 - F) + B_1 L_1 F / 2 + c_1], \end{aligned} \quad (A13)$$

$$\text{subject to:} \quad I + q_2 J F = \sum_i q_i [(A_i + c_i) L_i F - (A_i L_i - B_i L_i^2 / 2) F^2].$$

The constraint to ensure that the first stage of financing is always worthwhile becomes $I + q_2 J \leq c' + B' / 2 + [\max(0, A' - B' - c')]^2 / [2(2A' - B')]$, where $A' = L q_1 A_1 + q_2 A_2$, $B' = L^2 q_1 B_1 + q_2 B_2$, and $c' = L q_1 c_1 + q_2 c_2$.

We first consider the case of $A_1 = A_2 = I = 0$. The optimal milestone contract (A11) reduces to

$$\begin{aligned} \text{Max } \Phi_M &= \sum_i p_i [(B_i L_i / 2)(2f_i - f_i^2) + c_i] - p_2 J + Z p_1 f_1 [B_1 L_1 f_1 / 2 + c_1], \\ 0 \leq f_1 \leq f_2 \leq 1 \end{aligned} \quad (A14)$$

$$\text{subject to:} \quad q_2 J = \sum_i q_i [c_i L_i f_i + (B_i L_i^2 / 2) f_i^2],$$

and the round financing contract (A13) reduces to

$$\Phi_R = \sum_i p_i [(B_i L_i / 2)(2F - F^2) + c_i] - p_2 J + Z p_1 F [B_1 L_1 F / 2 + c_1], \quad (A15)$$

$$\text{subject to:} \quad q_2 J F = \sum_i q_i [c_i L_i F + (B_i L_i^2 / 2) F^2].$$

The round financing contract has $F = 0$. Therefore, $\Phi_R = \sum_i p_i c_i - p_2 J$. From (A14), a feasible milestone contract is $f_1 = 0$, f_2 satisfying $J = [c_2 f_2 + (B_2 / 2) f_2^2]$. Note that $f_2 \leq J / c_2 < 1$. This milestone contract yields an objective function $\Phi_M \geq \sum_i p_i c_i - p_2 J = \Phi_R$.

We next consider the case of $q_2 J = 0$. The optimal milestone contract (A11) reduces to

$$\begin{aligned} \text{Max } \Phi_M &= \sum_i p_i [(A_i / 2)(1 - f_i^2) + (B_i L_i / 2)(2f_i - f_i^2) + c_i] - (p_2 / q_2) I - p_2 J \\ 0 \leq f_1 \leq f_2 \leq 1 &\quad + Z p_1 f_1 [A_1 (1 - f_1) + B_1 L_1 f_1 / 2 + c_1], \end{aligned} \quad (A16)$$

$$\text{subject to:} \quad I = \sum_i q_i [(A_i + c_i) L_i f_i - (A_i L_i - B_i L_i^2 / 2) f_i^2],$$

and the round financing contract (A13) reduces to

$$\begin{aligned} \Phi_R &= \sum_i p_i [(A_i/2)(1 - F^2) + (B_i L_i/2)(2F - F^2) + c_i] - (p_2/q_2)I - p_2J \\ &\quad + Z p_1 F [A_1(1 - F) + B_1 L_1 F/2 + c_1], \end{aligned} \quad (A17)$$

subject to: $I = \sum_i q_i [(A_i + c_i) L_i F - (A_i L_i - B_i L_i^2/2) F^2]$.

Note that $f_1 = f_2 = F$ is a feasible milestone contract. Therefore, a milestone contract can replicate the round contract. ♣

Proof of Proposition 8. Suppose venture capitalist liquidity preference is at least as important as heterogeneity ($Z \leq 0$). When venture capitalist effort is relatively unimportant and the first stage financing requirement is relatively small (B_1 , B_2 , and I small), round financing remains preferred to milestone financing.

Let $B_1 = B_2 = 0$ and $I = 0$. The optimal milestone contract (A11) reduces to

$$\begin{aligned} \text{Max } \Phi_M &= \sum_i p_i [(A_i/2)(1 - f_i^2) + c_i] - p_2 J + Z p_1 f_1 [A_1(1 - f_1) + c_1], \\ 0 \leq f_1 \leq f_2 \leq 1 \end{aligned} \quad (A18)$$

subject to: $q_2 J = \sum_i q_i [(A_i + c_i) L_i f_i - (A_i L_i) f_i^2]$,

and the round financing contract (A13) reduces to

$$\begin{aligned} \Phi_R &= \sum_i p_i [(A_i/2)(1 - F^2) + c_i] - p_2 J + Z p_1 F [A_1(1 - F) + c_1], \\ \text{subject to: } q_2 J F &= \sum_i q_i [(A_i + c_i) L_i F - (A_i L_i) F^2]. \end{aligned} \quad (A19)$$

The round financing contract has $F = 0$. From (A19), this is first best. From the constraint in (A18), f_1 and f_2 are non-negative. If $q_2 J > 0$, at least one of f_1 and f_2 must be positive, either of which reduces (A18) below first best. ♣

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